Matematika 2,
vježbe, 20. 5. 2020.

* Neke specijalne matrice *

* **TRAG** matrice je zbroj elemenata matrice na glavnoj dijagonali.
Trag matrice A označavamo s tr A.

Zad Izračunajte trag sljedećih matrica:

a) $A = \begin{bmatrix} 44 & -15 \\ -3 & -9 \end{bmatrix}$ Rj $\text{tr}\,A = 44 + (-9) = 35$

b) $B = \begin{bmatrix} 9 & 13 & -54 \\ -3 & -18 & 4 \\ 4 & 75 & -9 \end{bmatrix}$ Rj $\text{tr}\,B = \cancel{9} - 18 - \cancel{9} = -18$

c) $C = \begin{bmatrix} -8 & 19 & -35 & 1 \\ 0 & 10 & -6 & 5 \\ 8 & 0 & -22 & 4 \\ -473 & -101 & 250 & 4 \end{bmatrix}$ Rj $\text{tr}\,C = -8 + 10 - 22 + 4 = -16$

* **TRANSPONIRANU** matricu matrice A označavamo s A^T te vrijedi $(A^T)^T = A$.

(Transponiranu matricu dobijemo tako da prvi red naše polazne matrice stavimo u prvi stupac, drugi red polazne matrice stavimo u drugi stupac itd.)

Matrica A je SIMETRIČNA ako je $A = A^T$.

Matrica A je ANTISIMETRIČNA ako je $A = -A^T$.

zad Transponirajte sljedeće matrice te provjerite jesu li simetrične, antisimetrične ili niti jedno od navedenog:

a) $$A = \begin{bmatrix} 5 & 2 & -6 \\ 2 & 3 & 1 \\ -6 & 1 & 4 \end{bmatrix}$$

Rj $$A^T = \begin{bmatrix} 5 & 2 & -6 \\ 2 & 3 & 1 \\ -6 & 1 & 4 \end{bmatrix}$$

Uočimo da je $A = A^T$ pa zaključujemo da je A simetrična matrica

b) $$B = \begin{bmatrix} 0 & 3 & 12 \\ -3 & 0 & 9 \\ -12 & -9 & 0 \end{bmatrix}$$

Rj $$B^T = \begin{bmatrix} 0 & -3 & -12 \\ 3 & 0 & -9 \\ 12 & 9 & 0 \end{bmatrix}$$

Vidimo da je $B^T \neq B$ dakle B nije simetrična!

$$-B^T = \begin{bmatrix} 0 & 3 & 12 \\ -3 & 0 & 9 \\ -12 & -9 & 0 \end{bmatrix}$$

$\Rightarrow$ $-B^T = B$ dakle, B je antisimetrična matrica

c) $$C = \begin{bmatrix} 3 & 2 & -9 \\ -18 & 1 & 0 \\ 32 & 8 & 0 \end{bmatrix}$$

Rj $$C^T = \begin{bmatrix} 3 & -18 & 32 \\ 2 & 1 & 8 \\ -9 & 0 & 0 \end{bmatrix}$$

$\Rightarrow$ $C^T \neq C$ pa C nije simetrična

$$-C^T = \begin{bmatrix} -3 & 18 & -32 \\ -2 & -1 & -8 \\ 9 & 0 & 0 \end{bmatrix}$$

$\rightarrow$ $-C^T \neq C$ pa C nije ni antisimetrična

Zad Dane su matrice

$$A=\begin{bmatrix}1 & -4 & -2\\ -4 & 8 & 0\\ -2 & 0 & -4\end{bmatrix},\ B=\underbrace{\begin{bmatrix}1 & 0 & 3\\ 0 & -1 & 2\\ 2 & 0 & 7\end{bmatrix}}_{trB=1-1+7=7}\ i\ C=\underbrace{\begin{bmatrix}1 & 0 & 3\\ 2 & -1 & 1\end{bmatrix}}_{(2\times3)}.$$

Izračunajte :

a) $$C^T\cdot C=\underbrace{\begin{bmatrix}1 & 2\\ 0 & -1\\ 3 & 1\end{bmatrix}}_{(3\times\cancel{2})\cdot(\cancel{2}\times3)=(3\times3)}\cdot\begin{bmatrix}1 & 0 & 3\\ 2 & -1 & 1\end{bmatrix}=\begin{bmatrix}5 & -2 & 5\\ -2 & 1 & -1\\ 5 & -1 & 10\end{bmatrix}$$

b) $trB\cdot I-C^T\cdot C+3\cdot B^T$

$$=7\cdot\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}-\begin{bmatrix}5 & -2 & 5\\ -2 & 1 & -1\\ 5 & -1 & 10\end{bmatrix}+3\cdot\begin{bmatrix}1 & 0 & 2\\ 0 & -1 & 0\\ 3 & 2 & 7\end{bmatrix}$$

$$=\begin{bmatrix}7 & 0 & 0\\ 0 & 7 & 0\\ 0 & 0 & 7\end{bmatrix}+\begin{bmatrix}-5 & 2 & -5\\ 2 & -1 & 1\\ -5 & 1 & -10\end{bmatrix}+\begin{bmatrix}3 & 0 & 6\\ 0 & -3 & 0\\ 9 & 6 & 21\end{bmatrix}$$

$$=\begin{bmatrix}5 & 2 & 1\\ 2 & 3 & 1\\ 4 & 7 & 18\end{bmatrix}$$

c) Provjerite je li matrica A simetrična, antisimetrična ili niti jedno od navedenog.

Rj: $A^T=\begin{bmatrix}1 & -4 & -2\\ -4 & 8 & 0\\ -2 & 0 & -4\end{bmatrix}\Rightarrow A^T=A$ pa je A SIMETRIČNA matrica

Regularne matrice

(udžbenik, str. 223)

Def Za kvadratnu matricu A kažemo da je REGULARNA ili INVERTIBILNA ako postoji kvadratna matrica B takva da je

$$A\cdot B = B\cdot A = I.$$

U suprotnom kažemo da je matrica A singularna.

Matricu B koja zadovoljava prethodnu definiciju označavamo s A^{-1} i nazivamo INVERZNOM MATRICOM matrice A. Prema tome imamo

$$A\cdot A^{-1} = A^{-1}\cdot A = I \quad (***).$$

Zad Gaussovom metodom nadite inverz sljedećih matrica:

a) $A = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$.

Rj Gaussovom metodom inverz tražimo tako da matricu A proširimo jediničnom matricom istog tipa kao što je naša polazna matrica A. $[A \,\vdots\, I]$. Cilj nam je na mjestu matrice A dobiti jediničnu matricu. Matrica koja se u tom trenutku nalazi na "desnoj strani" proširene matrice je traženi inverz matrice A.

$$[A \,\vdots\, I] = \left[\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 4 & 3 & 0 & 1 \end{array}\right] \begin{array}{l} /(-2) \\ + \end{array} \sim \left[\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{array}\right] \begin{array}{l} + \\ /(-1) \end{array}$$

$$\sim \left[\begin{array}{cc|cc} 2 & 0 & 3 & -1 \\ 0 & 1 & -2 & 1 \end{array}\right] \begin{array}{l} /:2 \\ \end{array} \sim \left[\begin{array}{cc|cc} 1 & 0 & 3/2 & -1/2 \\ 0 & 1 & -2 & 1 \end{array}\right]$$

$$\underbrace{\begin{matrix} 3/2 & -1/2 \\ -2 & 1 \end{matrix}}_{A^{-1}}$$

$$A^{-1} = \begin{bmatrix} 3/2 & -1/2 \\ -2 & 1 \end{bmatrix}$$

DZ: provjerite da vrijedi $A \cdot A^{-1} = A^{-1} \cdot A = I$.

b) $$B = \begin{bmatrix} -1 & 3 \\ 5 & -6 \end{bmatrix}$$

Rj $$[B|I] = \left[\begin{array}{cc|cc} -1 & 3 & 1 & 0 \\ 5 & -6 & 0 & 1 \end{array}\right] \begin{array}{l} |\cdot 5 \\ \swarrow + \end{array} \sim \left[\begin{array}{cc|cc} -1 & 3 & 1 & 0 \\ 0 & 9 & 5 & 1 \end{array}\right] \begin{array}{l} \\ |:3 \end{array}$$

$$\sim \left[\begin{array}{cc|cc} -1 & 3 & 1 & 0 \\ 0 & 3 & 5/3 & 1/3 \end{array}\right] \begin{array}{l} \nwarrow + \\ |(-1) \end{array} \sim \left[\begin{array}{cc|cc} -1 & 0 & -2/3 & -1/3 \\ 0 & 3 & 5/3 & 1/3 \end{array}\right] \begin{array}{l} |\cdot(-1) \\ |:3 \end{array}$$

$$\sim \underbrace{\left[\begin{array}{cc|cc} 1 & 0 & 2/3 & 1/3 \\ 0 & 1 & 5/9 & 1/9 \end{array}\right]}_{B^{-1}} \Rightarrow B^{-1} = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{5}{9} & \frac{1}{9} \end{bmatrix}$$

c) $$A = \begin{bmatrix} 1 & 1 & -2 \\ 0 & -1 & 1 \\ 1 & -2 & 0 \end{bmatrix}$$

Rj $$[A|I] = \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 1 & -2 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} |(-1) \\ \downarrow \\ \swarrow + \end{array} \sim \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & -3 & 2 & -1 & 0 & 1 \end{array}\right] \begin{array}{l} \\ |(-3) \\ \swarrow + \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 & -3 & 1 \end{array}\right] \begin{array}{l} \\ \nwarrow + \\ |\cdot 1 \end{array} \sim \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 1 & 0 & 0 \\ 0 & -1 & 0 & -1 & -2 & 1 \\ 0 & 0 & -1 & -1 & -3 & 1 \end{array}\right] \begin{array}{l} \nwarrow + \\ |\cdot 1 \\ \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 0 & -2 & 1 \\ 0 & -1 & 0 & -1 & -2 & 1 \\ 0 & 0 & -1 & -1 & -3 & 1 \end{array}\right] \begin{array}{l} \nwarrow \\ + \\ |(-2) \end{array} \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 4 & -1 \\ 0 & -1 & 0 & -1 & -2 & 1 \\ 0 & 0 & -1 & -1 & -3 & 1 \end{array}\right] \begin{array}{l} \\ |\cdot(-1) \\ |\cdot(-1) \end{array}$$

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$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 4 & -1 \\ 0 & 1 & 0 & 1 & 2 & -1 \\ 0 & 0 & 1 & 1 & 3 & -1 \end{array}\right] \quad \Rightarrow \quad A^{-1} = \underline{\underline{\begin{bmatrix} 2 & 4 & -1 \\ 1 & 2 & -1 \\ 1 & 3 & -1 \end{bmatrix}}}$$

$$\underbrace{\begin{matrix} 2 & 4 & -1 \\ 1 & 2 & -1 \\ 1 & 3 & -1 \end{matrix}}_{A^{-1}}$$

d) $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & -3 \\ 1 & 1 & -1 \end{bmatrix}$

Rj $[B \mid I] = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 0 & -3 & 0 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 & 1 \end{array}\right] \begin{matrix} |(-2) \\ \hookleftarrow + \\ \end{matrix} \sim \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -4 & -9 & -2 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 & 1 \end{array}\right] \begin{matrix} |(-1) \\ + \\ \hookleftarrow \end{matrix}$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -4 & -9 & -2 & 1 & 0 \\ 0 & -1 & -4 & -1 & 0 & 1 \end{array}\right] \begin{matrix} \\ \hookleftarrow + \\ |\cdot(-4) \end{matrix} \sim \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 0 & 7 & 2 & 1 & -4 \\ 0 & -1 & -4 & -1 & 0 & 1 \end{array}\right] \begin{matrix} \hookleftarrow \\ + \\ |\cdot 2 \end{matrix}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -5 & -1 & 0 & 2 \\ 0 & 0 & 7 & 2 & 1 & -4 \\ 0 & -1 & -4 & -1 & 0 & 1 \end{array}\right] \begin{matrix} \\ |:7 \\ \end{matrix} \sim \left[\begin{array}{ccc|ccc} 1 & 0 & -5 & -1 & 0 & 2 \\ 0 & 0 & 1 & 2/7 & \frac{1}{7} & -\frac{4}{7} \\ 0 & -1 & -4 & -1 & 0 & 1 \end{array}\right] \begin{matrix} \hookleftarrow + \\ |\cdot 5 \\ \end{matrix}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{3}{7} & \frac{5}{7} & -\frac{6}{7} \\ 0 & 0 & 1 & 2/7 & \frac{1}{7} & -\frac{4}{7} \\ 0 & -1 & -4 & -1 & 0 & 1 \end{array}\right] \begin{matrix} \\ |4 \\ \hookleftarrow + \end{matrix} \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{3}{7} & \frac{5}{7} & -\frac{6}{7} \\ 0 & 0 & 1 & 2/7 & 1/7 & -4/7 \\ 0 & -1 & 0 & 7/7 & 4/7 & -9/7 \end{array}\right] \begin{matrix} \\ \\ |\cdot(-1) \end{matrix}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{3}{7} & \frac{5}{7} & -\frac{6}{7} \\ 0 & 0 & 1 & 2/7 & \frac{1}{7} & -\frac{4}{7} \\ 0 & 1 & 0 & -1/7 & -\frac{4}{7} & \frac{9}{7} \end{array}\right] \begin{matrix} \\ \updownarrow \\ \end{matrix} \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{3}{7} & \frac{5}{7} & -\frac{6}{7} \\ 0 & 1 & 0 & -1/7 & -4/7 & 9/7 \\ 0 & 0 & 1 & 2/7 & 1/7 & -4/7 \end{array}\right]$$

$$\underbrace{\begin{matrix} \frac{3}{7} & \frac{5}{7} & -\frac{6}{7} \\ -1/7 & -4/7 & 9/7 \\ 2/7 & 1/7 & -4/7 \end{matrix}}_{B^{-1}}$$

e) $C = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}$

Rj $[C \,|\, I] = \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 2 & -1 & 3 & 0 & 1 & 0 \\ 4 & 1 & 8 & 0 & 0 & 1 \end{array}\right] \begin{matrix} |\cdot(-2) \\ \leftarrow + \\ \end{matrix} \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & -1 & -1 & -2 & 1 & 0 \\ 4 & 1 & 8 & 0 & 0 & 1 \end{array}\right] \begin{matrix} |\cdot(-4) \\ \\ \leftarrow + \end{matrix}$

$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & -1 & -1 & -2 & 1 & 0 \\ 0 & 1 & 0 & -4 & 0 & 1 \end{array}\right] \begin{matrix} \\ \leftarrow + \\ |\cdot 1 \end{matrix} \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & -1 & -6 & 1 & 1 \\ 0 & 1 & 0 & -4 & 0 & 1 \end{array}\right] \begin{matrix} \leftarrow + \\ |\cdot 2 \\ \end{matrix}$

$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -11 & 2 & 2 \\ 0 & 0 & -1 & -6 & 1 & 1 \\ 0 & 1 & 0 & -4 & 0 & 1 \end{array}\right] \begin{matrix} \\ |\cdot(-1) \\ \end{matrix} \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -11 & 2 & 2 \\ 0 & 0 & 1 & 6 & -1 & -1 \\ 0 & 1 & 0 & -4 & 0 & 1 \end{array}\right] \begin{matrix} \\ \updownarrow \\ \end{matrix}$

$\sim \left[\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \middle| \underbrace{\begin{matrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{matrix}}_{C^{-1}} \right].$ $\quad C^{-1} = \begin{bmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{bmatrix}$

f) $D = \begin{bmatrix} 0 & -1 & 1 \\ 1 & -2 & 5 \\ 2 & 1 & 4 \end{bmatrix}$ (DZ)

Rj $D^{-1} = \begin{bmatrix} 13 & -5 & 3 \\ -6 & 2 & -1 \\ -5 & 2 & -1 \end{bmatrix}$

RANG MATRICE

Def Maksimalan broj linearno nezavisnih stupaca matrice A zovemo rang stupaca matrice A, a maksimalan broj linearno nezavisnih redaka zovemo rang redaka matrice A.

* Elementarnim operacijama nad recima neke matrice (kao i elementarnim operacijama nad njenim stupcima) ne mijenjaju se ni rang stupaca ni rang redaka.

* Rang stupaca matrice A jednak je rangu redaka matrice A. Taj broj zove se RANG MATRICE A i oznacava s r(A).

Zad Odredite rang sljedecih matrica:

a) $$A = \begin{bmatrix} 1 & -4 & -2 \\ -4 & 8 & 0 \\ -2 & 0 & -4 \end{bmatrix} \begin{matrix} |\cdot 4 \\ \, \\ \, \end{matrix} \sim \begin{bmatrix} 1 & -4 & -2 \\ 0 & -8 & -8 \\ -2 & 0 & -4 \end{bmatrix} \begin{matrix} |\cdot 2 \\ \, \\ \, \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -4 & -2 \\ 0 & -8 & -8 \\ 0 & -8 & -8 \end{bmatrix} \begin{matrix} \, \\ |(-1) \\ \, \end{matrix} \sim \begin{bmatrix} 1 & -4 & -2 \\ 0 & -8 & -8 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} \, \\ |:(-8) \\ \, \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -4 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} \, \\ |\cdot 4 \\ \, \end{matrix} \sim \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \qquad \underline{\underline{r(A) = 2}}$$

$$\begin{matrix} /(-2) \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$/(-1)$$

b) $$B=\begin{bmatrix}2 & 1 & 3 & -2\\ -3 & 2 & 1 & -3\\ -4 & 5 & 5 & -8\end{bmatrix}\begin{matrix}/\cdot 2\\ \\ +\end{matrix} \sim \begin{bmatrix}2 & 1 & 3 & -2\\ -3 & 2 & 1 & -3\\ 0 & 7 & 11 & -12\end{bmatrix}\begin{matrix}/\cdot 2\\ +\\ \end{matrix}$$

$$\sim \begin{bmatrix}2 & 1 & 3 & -2\\ 1 & 4 & 7 & -7\\ 0 & 7 & 11 & -12\end{bmatrix}\begin{matrix}+\\ /(-2)\\ \end{matrix} \sim \begin{bmatrix}0 & -7 & -11 & 12\\ 1 & 4 & 7 & -7\\ 0 & 7 & 11 & -12\end{bmatrix}$$

$/(-4)$ +

$/(-7)$ +

$/7$ +

$$\sim \begin{bmatrix}0 & -7 & -11 & 12\\ 1 & 0 & 0 & 0\\ 0 & 7 & 11 & -12\end{bmatrix}\begin{matrix}/\cdot 1\\ \\ +\end{matrix} \sim \begin{bmatrix}0 & -7 & -11 & 12\\ 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}/:(-7)$$

$$\sim \begin{bmatrix}0 & 1 & 11/7 & -12/7\\ 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix} \sim \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 11/7 & -12/7\\ 0 & 0 & 0 & 0\end{bmatrix} \quad \underline{\underline{r(B)=2}}$$

Možemo još "uništiti" $11/7$ i $-12/7$

c) $$C=\begin{bmatrix}2 & -1 & 3\\ 3 & -2 & 1\\ 5 & -3 & 4\end{bmatrix}\begin{matrix}/\cdot(-2)\\ \\ +\end{matrix} \sim \begin{bmatrix}2 & -1 & 3\\ 3 & -2 & 1\\ 1 & -1 & -2\end{bmatrix}\begin{matrix}\\ +\\ /(-3)\end{matrix} \sim \begin{bmatrix}2 & -1 & 3\\ 0 & 1 & 7\\ 1 & -1 & -2\end{bmatrix}\begin{matrix}+\\ \\ /\cdot(-2)\end{matrix}$$

$$\sim \begin{bmatrix}0 & 1 & 7\\ 0 & 1 & 7\\ 1 & -1 & -2\end{bmatrix}\begin{matrix}+\\ /(-1)\\ \end{matrix} \sim \begin{bmatrix}0 & 0 & 0\\ 0 & 1 & 7\\ 1 & -1 & -2\end{bmatrix}\begin{matrix}\\ /\cdot 1\\ +\end{matrix} \sim \begin{bmatrix}0 & 0 & 0\\ 0 & 1 & 7\\ 1 & 0 & 5\end{bmatrix}$$

$/(-7)$ +

$$\sim \begin{bmatrix}0 & 0 & 0\\ 0 & 1 & 0\\ 1 & 0 & 5\end{bmatrix} \sim \begin{bmatrix}0 & 0 & 0\\ 0 & 1 & 0\\ 1 & 0 & 0\end{bmatrix} \sim \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{bmatrix} \quad \underline{\underline{r(C)=2}}$$

$/(-5)$ +

d) $$D = \begin{bmatrix} 3 & 1 & 2 \\ -6 & -2 & -4 \\ -3 & -1 & -2 \end{bmatrix} \begin{matrix} |\cdot 1 \\ \\ \hookleftarrow \end{matrix} + \sim \begin{bmatrix} 3 & 1 & 2 \\ -6 & -2 & -4 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} |\cdot 2 \\ \hookleftarrow \\ \\ \end{matrix} +$$

$$\sim \begin{bmatrix} 3 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \underline{\underline{r(D) = 1}}$$